

2021

B.A./B.Sc.(General)-5th Semester

Mathematics

Paper-I: Analysis-I

Time allowed: 3 Hours

Max. Marks: 30

NOTE: Attempt five questions in all, selecting atleast two questions from each Unit.

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UNIT - I

- I. (a) Prove that union of finite number of countable sets is a countable set.
- (b) Let f be the function defined on $[0,1]$ as $f(x) = \begin{cases} 1+x & \text{if } x \neq \frac{1}{2} \\ 0 & \text{if } x = \frac{1}{2} \end{cases}$. Show that f is Riemann integrable on $[0,1]$ and $\int_0^1 f(x)dx = \frac{3}{2}$. (3+3)
- II. (a) Let $f(x) = x^2$ and $P = \left\{ -\frac{3}{2}, -\frac{1}{2}, \frac{1}{4}, 1 \right\}$. Evaluate $L(P,f)$.
- (b) State and prove Darboux's Theorem. (3+3)
- III. (a) Let f be a continuous function defined on $[a,b]$ and let $F(x) = \int_a^x f(t)dt \forall x \in [a,b]$ then prove that $\frac{d}{dx}(F(x)) = f(x) \forall x \in [a,b]$.
- (b) Let $f(x) = \begin{cases} 1-x & \text{if } x \text{ is irrational} \\ \sqrt{1-x^2} & \text{if } x \text{ is rational} \end{cases}$. Show that f is Riemann integrable on $[0,1]$. (3+3)
- IV. (a) Show that $\int_0^1 \frac{(1-x^4)^{\frac{3}{4}}}{(1+x^4)^2} dx = \frac{1}{4(2)^{\frac{1}{4}}} B\left(\frac{7}{4}, \frac{1}{4}\right)$ where $B(m,n)$ is the Beta function.
- (b) Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx = \frac{\pi}{2\sqrt{2}} \cdot \int_0^\infty \frac{1}{\sqrt{1+x^4}} dx$. (3+3)

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UNIT-II

- V. (a) Show that $\int_0^{\infty} \left(\frac{1}{1+x} - e^{-x} \right) \frac{dx}{x}$ is convergent.
- (b) Discuss the convergence of $\int_0^1 \left(\log \frac{1}{x} \right)^m dx$. (3+3)
- VI. (a) Discuss the convergence of $\int_0^1 \frac{(x^m + x^{-m})}{x} \log(1+x) dx$.
- (b) Show that $\int_0^{\infty} \frac{\sin x}{x} dx$ is not absolutely convergent. (3+3)
- VII. (a) Show that $\int_0^{\pi/2} \sin x \log(\sin x) dx$ is convergent with value $\log\left(\frac{2}{e}\right)$.
- (b) If $|a| < 1$ Evaluate $\int_0^{\pi} \frac{\log(1+a \cos x)}{\cos x} dx$ (3+3)
- VIII. (a) Evaluate $\int_0^{\pi/2} \log(\alpha^2 \cos^2 \theta + \beta^2 \sin^2 \theta) d\theta$ where $\alpha > 0$, $\beta > 0$.
- (b) If $f(x)$ is a continuous function on $(0, \infty)$ having points of infinite discontinuity at 0 and ∞ only, $\lim_{x \rightarrow 0^+} f(x) = f_0$ and $\lim_{x \rightarrow \infty} f(x) = f_1$ then prove that $\int_0^{\infty} \frac{f(ax) - f(bx)}{x} dx = (f_0 - f_1) \log \frac{b}{a}$. (3+3)

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